Particle-Flux Separation and Quasi-excitations in Quantum Hall Systems

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The quasiexcitations of quantum Hall systems at the filling factor $\nu = p/(2pq\pm1)$ are studied in terms of chargeon and fluxon introduced previously as constituents of an electron at $\nu = 1/2$. At temperatures $T < T_{\rm PFS}(\nu)$, the phenomenon so-called particle-flux separation takes place, and chargeons and fluxons are deconfined to behave as quasiparticles. Bose condensation of fluxons justify the (partial) cancellation of external magnetic field. Fluxons describe correlation holes, while chargeons describe composite fermions. They contribute to the resistivity $\rho_{xy} = h/(\nu e^2)$ additively.

KEYWORDS: quantum Hall effect, composite fermions, gauge field theory, confinement-deconfinement transition

In the last decade, the quasi-excitations in a two-dimensional electron system in a strong magnetic field have been intensively studied. Currently, composite fermions[1)] (CF) for the filling factors $\nu = p/(2pq+1)$ (p, q; positive integers) and composite bosons[2,3,4)] (CB) for $\nu = 1/(2q+1)$ play very important roles in a unified view of the quantum Hall effect (QHE), and the Chern-Simons (CS) gauge theory is often used to describe them. The validity of these approaches relies upon the possibility that the CS fluxes attached to each CF or CB cancel out the magnetic field, partly in the case of CF, and completely in the case of CB (and CF at $\nu = 1/2$), as assumed in the mean-field theory (MFT).

In ref.,[5,6)] we considered the system of $\nu = 1/2$ and argued that this cancellation really takes place at temperatures T below a certain critical temperature $T_{\rm PFS}$ via the mechanism which we call particle-flux separation (PFS). In ref.[6)] we introduced the chargeon and fluxon operators to describe the charge and flux degrees of freedom of an electron, respectively. They are confined within electrons by a U(1) gauge field at $T > T_{\rm PFS}$, and PFS is characterized as a deconfinement phenomenon of the gauge dynamics of this gauge field. The PFS is a counterpart of the charge-spin separation (CSS)[7,8)] in high-T_C cuprates, and justifies the CF picture of Jain,[1)] Halperin et al.,[9)] and other works.

In recent years, some important studies of the CS gauge theory of the QHE have appeared. Shankar and Murthy[10)] studied the quasi-excitations in terms of a CF(CB) operator and the longitudinal (or time)-component of the CS gauge field which becomes dynamical because of the enlargement of the Hilbert space. However, their CF(CB) operator *does not commute* with the CS field and simple decoupling as they described *does not hold*. Stern et al.[11)] also studied the transport properties of CFs at $\nu = 1/2$, but the CS constraint is not respected.

Based on these studies, the treatments of CS constraint and the associated ambiguity of independent variables certainly require further investigations in order to establish a consistent theory of CFs. In this letter we address this problem by generalizing the chargeon and fluxon approach of ref.[6)] to CFs at $\nu = p/(2pq \pm 1)$. The lattice regularization and the second-quantized operators make it possible to study PFS nonperturbatively and identify quasi-excitations in a consistent manner. Furthermore, the transport properties are described quite neatly.

Let us consider the system of electrons on a two-dimensional lattice in an external uniform magnetic field $B^{\rm ex}$ in the transverse direction, and start with the following CS representation of the electron annihilation operator C_x at the site x,[5,6)]

$$C_x = \exp\left[2iq\sum_y \theta_{xy}\psi^\dagger_y\psi_y\right]\psi_x, \tag{1}$$

where θ_{xy} is the multivalued angle function on a lattice, and ψ_x is the fermionic annihilation operator of a so-called CS fermion. The phase factor describes that each electron carries $2q$ (q: positive integer) units of CS flux quanta. The filling factor is given by $\nu = 2\pi n/(eB^{\rm ex}a^2)$ ($\hbar = c = 1$), where $n \equiv \langle\psi^\dagger_x\psi_x\rangle$ is the average number of electrons or ψ_x's *per site* (note that $C^\dagger_x C_x = \psi^\dagger_x\psi_x$), and a is the lattice spacing. We choose $a \simeq \ell$ where $\ell = (eB^{\rm ex})^{-1/2}$ is the magnetic length. Hereafter, we set $a = 1$ in the formulae. If $B^{\rm ex}$ is partly cancelled by the CS fluxes uniformly as in MFT, each ψ_x feels the residual constant magnetic field $\Delta B \equiv B^{\rm ex} - 4\pi qn/e$. Then, FQHE takes place as the IQHE of ψ_x in ΔB at $1/\nu - 2q = \pm1/p$ (p: positive integer), or $\nu = p/(2pq\pm1)$. Below we see that this idea is actually realized by PFS of chargeons and fluxons.

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The Hamiltonian in terms of ψ_x is given by

$$H_\psi = -\frac{1}{2m}\sum_{x,j}\Big(\psi^\dagger_{x+j}\exp[i(A^{\rm CS}_{xj}-eA^{\rm ex}_{xj}-ea_{xj})]\psi_x + {\rm H.c.}\Big) + H_{\rm int}(\psi^\dagger_x\psi_x),$$
$$A^{\rm CS}_{xj} = 2q\epsilon_{ji}\sum_y \nabla_i\theta_{xy}\psi^\dagger_y\psi_y, \qquad (2)$$

where m is the effective electron mass, $j=1,2$ is the direction index (and the unit vectors), and $A^{\rm CS}_{xj}$ is the CS gauge field, its strength being $B^{\rm CS}_x \equiv \epsilon_{ij}\nabla_i A^{\rm CS}_{xj} = 4\pi q\psi^\dagger_x\psi_x$. $H_{\rm int}$ represents replusive Coulombic interaction between electrons or ψ_x, and $A^{\rm ex}_{xj}$ is the electromagnetic (EM) potential for $B^{\rm ex}$, $\epsilon_{ij}\nabla_i A^{\rm ex}_{xj} = B^{\rm ex}$. To study the EM response functions, we introduced an EM source potential a_{xj}.

Let us introduce the chargeon η_x and fluxon ϕ_x operators through

$$\psi_x = \phi_x\eta_x, \qquad (3)$$

which shows that ψ_x is a composite of a chargeon and a fluxon. We quantize the chargeon η_x as a fermion, and the fluxon ϕ_x as a boson.[12)] From eq.(3), it is obvious that ψ_x and H_ψ are invariant under the U(1) "local gauge transformation",

$$(\eta_x,\phi_x)\to(e^{i\alpha_x}\eta_x, e^{-i\alpha_x}\phi_x) \text{ for each } x. \qquad (4)$$

To maintain equivalence with eq.(2) we impose the following local constraint:

$$\eta^\dagger_x\eta_x = \phi^\dagger_x\phi_x. \qquad (5)$$

Thus, the relations are $|0\rangle_\psi = |0\rangle_{\eta\phi}$, $\psi^\dagger_x|0\rangle_\psi = \eta^\dagger_x\phi^\dagger_x|0\rangle_{\eta\phi}$ at each x, where $|0\rangle_{\eta\phi} = |0\rangle_\eta|0\rangle_\psi$ $(\eta_x|0\rangle_\eta = \psi_x|0\rangle_\psi = 0)$.

The electron operator is expressed as

$$C_x = \exp\Big[2iq\sum_y\theta_{xy}\phi^\dagger_y\phi_y\Big]\phi_x\eta_x. \qquad (6)$$

From eq.(6) an electron is composed of a fluxon ϕ_x, a chargeon η_x, and $2q$ units of CS flux quanta generated by fluxons. This is illustrated in Fig.1.

H_ψ of eq.(2) is rewritten in terms of η_x and ϕ_x as

$$H_{\eta\phi} = -\frac{1}{2m}\sum_{x,j}\Big(\eta^\dagger_{x+j}\phi^\dagger_{x+j}W_{x+j}M_{x+j}M^\dagger_xW^\dagger_xe^{-iea_{xj}}\phi_x\eta_x + h.c.\Big) - \sum_x(\mu_\eta\eta^\dagger_x\eta_x + \mu_\phi\phi^\dagger_x\phi_x) + H_{\rm int}(\eta^\dagger_x\phi^\dagger_x\phi_x\eta_x),$$
$$W_x = \exp\Big[2iq\sum_y\theta_{xy}(\phi^\dagger_y\phi_y - n)\Big],$$
$$M_x = \exp\Big[i\sum_y\theta_{xy}n\big(2q-\frac{1}{\nu}\big)\Big]. \qquad (7)$$

We have added the terms with the chemical potentials μ_η,μ_ϕ to enforce $\langle\eta^\dagger_x\eta_x\rangle = \langle\phi^\dagger_x\phi_x\rangle = n$. (Note that $\psi^\dagger_x\psi_x = \phi^\dagger_x\phi_x = \eta^\dagger_x\eta_x$.)

We are interested in the low-energy dynamics, particularly how the local gauge symmetry (eq.(4)) and the constraint (5) are reflected there. We employ the path-integral formalism and respect the constraint (5) by introducing the Lagrange multiplier field λ_x. After decoupling $H_{\eta\phi}$ by introducing a complex auxiliary field V_{xj} on the link $(x,x+j)$, the Lagrangian is expressed as

$$L = -\sum_x\eta^\dagger_x(\partial_\tau - i\lambda_x - \mu_\eta)\eta_x - \sum_x\phi^\dagger_x(\partial_\tau + i\lambda_x - \mu_\phi)\phi_x + \frac{1}{2m}\sum_{x,j}\Big(V_{xj}J_{xj} + {\rm H.c.}\Big) - \frac{1}{2m}\sum_{x,j}|V_{xj}|^2 - H_4(\eta_x,\phi_x) - H_{\rm int}(\eta^\dagger_x\eta_x\phi^\dagger_x\phi_x),$$
$$H_4 \equiv \sum_{x,j}\Big(\frac{\gamma^2}{2m}\phi^\dagger_{x+j}\phi_x\phi^\dagger_x\phi_{x+j} + \frac{1}{2m\gamma^2}\eta^\dagger_{x+j}\eta_x\eta^\dagger_x\eta_{x+j}\Big)$$
$$J_{xj} \equiv \gamma\phi^\dagger_xW_xe^{ieca_{xj}}W^\dagger_{x+j}\phi_{x+j} + \frac{1}{\gamma}\eta^\dagger_{x+j}M_{x+j}e^{-ie(1-c)a_{xj}}M^\dagger_x\eta_x, \qquad (8)$$

where $\tau\in[0,\beta=1/(k_{\rm B}T)]$ is the imaginary time, and γ is a parameter which measures the ratio of the masses of chargeon and fluxon. c is an arbitrary constant, which appears in the EM charges of ϕ_x and η_x,

$$Q_\phi = ce,\ Q_\eta = (1-c)e. \qquad (9)$$

We shall discuss this important arbitrariness later. From eq.(8), $A_{x0}\equiv\lambda_x$ and A_{xj} of $V_{xj}\equiv|V_{xj}|\exp(iA_{xj})$ can be regarded as the time and spatial components of a U(1) gauge field $A_{x\mu}$. The system has a full U(1) gauge invariance under $A_{x\mu}\to A_{x\mu}+\nabla_\mu\alpha_x$ $(\nabla_0\equiv\partial/\partial\tau)$ and eq.(4) with τ-*dependent* α_x. There are no kinetic terms or Maxwell term of $A_{x\mu}$ in eq.(8). However, at low energies, $A_{x\mu}$ becomes dynamical as a result of "renormalization" (radiative corrections) by high-energy modes. At low energies, there are two possible realizations of the gauge dynamics: (i) a deconfinement phase where the fluctuations of $A_{x\mu}$ are weak, and chargeons and fluxons are deconfined and behave as quasi-free particles, or (ii) a confinement phase where the fluctuations are strong and chargeons and fluxons are confined into ψ_x, i.e., into the original electrons. The PFS is nothing but the deconfinement phenomenon (i), as we shall see below.

To induce the PFS, the repulsive Coulombic interaction $H_{\rm int}$ between electrons plays an important role. To clarify this, let us first focus on its short-range (i.e., nearest-neighbor) part by setting $H_{\rm int}(\psi^\dagger_x\psi_x) = g\sum\psi^\dagger_{x+j}\psi_{x+j}\psi^\dagger_x\psi_x$ with the coupling constant $g\ (>0)$. It is natural to estimate g as $g\simeq e^2/(\epsilon\ell)$, where ϵ is the dielectric constant. Because $\psi^\dagger_{x+j}\psi_{x+j}\psi^\dagger_x\psi_x = \eta^\dagger_{x+j}\eta_{x+j}\eta^\dagger_x\eta_x = \phi^\dagger_{x+j}\phi_{x+j}\phi^\dagger_x\phi_x$ by eq.(5), $H_{\rm int}$ may be rewritten at low energies as

$$H_{\rm int} = g_1\sum_{x,j}\eta^\dagger_{x+j}\eta_{x+j}\eta^\dagger_x\eta_x + g_2\sum_{x,j}\phi^\dagger_{x+j}\phi_{x+j}\phi^\dagger_x\phi_x, \qquad (10)$$

where $g_1+g_2=g$. Each term $H_{\rm int}$ or H_4 of eq.(8) is difficult to respect nonperturbatively, but when they are combined, one can treat them as irrelevant terms. In fact, we fix the values of g_1,g_2,γ by requiring that $H_{\rm int}$ and H_4 cancel out, $H_4+H_{\rm int}=0$, i.e., $g_1 = 1/(2m\gamma^2), g_2 = -\gamma^2/(2m)$. This choice reflects the idea that the fluxons and chargeons should behave as freely

as possible since they are candidates for quasi-excitations in the PFS state *at low energies*.

Let us put $V_{xj} = V_0 U_{xj}$ where U_{xj} is a $U(1)$ variable and V_0 is the expectation value of $|V_{xj}|$ by ignoring its fluctuations. We discuss the estimation of V_0 later. The effective action $S_{\rm eff}$ of $A_{x\mu}$ at low energies is then obtained by integrating out η_x, ϕ_x. We use the temporal gauge. At $T=0$, one can set $\lambda_x = 0$. However, at finite T, the zero modes of $\lambda_x(\tau)$, $\theta_x \equiv \beta^{-1}\int d\tau \lambda_x$, remain as integration variables in general. Thus

$$\int [d\eta][d\phi][d\theta] \exp\Big(\int_0^\beta d\tau L\Big) = \exp(-S_{\rm eff}). \quad (11)$$

To study PFS, we use the hopping expansion in powers of $V_0 U_{xj}$. The calculations are made straightforward by employing the single-site propagators like $\langle \eta_x(\tau_1)\eta_y^\dagger(\tau_2)\rangle = \delta_{xy} f_\eta(\tau_1-\tau_2)$ as utilized in ref.[5,6] The θ_x-integral in (11) takes the form,

$$\int [d\theta] \exp\Big(\sum_x \ln\frac{1+e^{\beta\mu_\eta + i\theta_x}}{1-e^{\beta\mu_\phi - i\theta_x}} + O(V_0^2)\Big). \quad (12)$$

From this integrand, we find that the ground state of θ_x is given by $\theta_x = 0$ (mod(2π)). The term of $O(V_0^2)$ in the exponent of eq.(12) is expanded around $\theta_x = 0$ as $-bV_0^2\sum_{x,j}(\nabla_j\theta_x)^2$ where b is a positive function of U_{xj}. This assures that $\theta_x = 0$ is stable. The excitation modes of θ_x are massive and the time component of $A_{x\mu}$ is screened, hence the perturbative calculations which assume the small fluctuations of λ_x are justified. The constraint (5) becomes irrelevant at low energies. Therefore, we set $\lambda_x = 0$ in L to obtain

$$\begin{aligned} S_{\rm eff} &= S_0 + S_2 + O(V_0^4), \\ S_2 &= V_0^2 \sum_{x,j}\Big[\frac{\beta}{2m} - \frac{n(1-n)}{4m^2}(\gamma^2+\gamma^{-2})\beta^2 U^\dagger_{xj,0}U_{xj,0}\Big], \\ U_{xj,0} &\equiv \frac{1}{\beta}\int_0^\beta d\tau U_{xj}(\tau). \end{aligned} \quad (13)$$

The properties of the quasi-excitations, i.e., whether PFS takes place or not, depend on the behavior of U_{xj}. From S_2 of eq.(13), it is obvious that at large β, i.e., at low T, $U_{xj,0}$ dominates at $|U_{xj,0}| \simeq 1$ and the fluctuations of A_{xj} are strongly suppressed. In $O(V_0^4)$ of $S_{\rm eff}$, plaquette terms (magnetic terms) like $U_{x2,0}U_{x+2,1,0}U^\dagger_{x+1,2,0}U^\dagger_{x1,0}$ appear, and their coefficients also become large at low T. Therefore, at low T, A_{xj} is in a deconfinement phase and the PFS occurs. Perturbative calculations with respect to A_{xj} are justified. The "transition temperature" $T_{\rm PFS}$ is estimated by setting the coefficient of $|U_{xj,0}|^2$ in S_2 at unity,[5,6]

$$V_0^2(T_{\rm PFS})\frac{n(1-n)}{4m^2k_{\rm B}^2T_{\rm PFS}^2}(\gamma^2+\gamma^{-2}) \simeq 1. \quad (14)$$

The analysis developed in lattice gauge theory predicts that the phase transition at $T_{\rm PFS}$ is smooth, as in CSS,[7,8] so our hopping expansion of $S_{\rm eff}$ in powers of V_0U_{xj} is justified a posteriori. It corresponds to the Ginzburg-Landau theory of global symmetry. This is in sharp contrast to most other studies of CS gauge theories working in the continuum.

Numerical estimation of $T_{\rm PFS}$ is given from eq.(14) for $\nu = 1/2$ by calculating $V_0^2(T)$ in a MFT of eq.(8) obtained by setting $\phi_x = \sqrt{n}, \lambda_x = 0$[13] as

$$T_{\rm PFS} = 4 \sim 4.5{\rm K} \quad \text{for} \quad g = (0.1 \sim 1)\times\frac{e^2}{\epsilon\ell}, \quad (15)$$

where $a = \ell$, $B^{\rm ex} = 10[{\rm T}], m = 0.067\, m_{\rm electron}, \epsilon = 13$. Then $\gamma = 0.96 \sim 0.69$ and the masses of chargeon and fluxon at $T=0$ are $m_\eta \equiv \gamma V_0^{-1}m = (6.5 \sim 4.7)m, m_\phi \equiv \gamma^{-1}V_0^{-1}m = (7.1 \sim 9.9)m$. $T_{\rm PFS}$ of eq.(15) seems consistent with the experimental results.[14] The highest temperature $T_{\rm BC}$ at which FQHE is observed is lower than $T_{\rm PFS}$ since FQHE is due to the Bose condensation of fluxons, as we shall see below.

We have obtained the above confinement-deconfinement phase transition (CDPT) by using techniques of lattice gauge theory. One may wonder if this transition survives in the "continuum limit". The CDPT at finite T was first discovered by Polyakov[15] and Susskind[16] in lattice gauge theory. After that, more detailed investigations, including numerical studies and renormalization-group (RG) analyses, confirmed the existence of this CDPT in the continuum. The lattice models are regarded in these cases as effective models of RG, and the transition temperature is a RG-invariant quantity.

In the PFS states, one may neglect fluctuations of U_{xj} as the first approximation. Then, the ground state of electrons $|G\rangle_C$ is given by the product $|G\rangle_C = |G\rangle_\phi |G\rangle_\eta$, where $|G\rangle_{\phi(\eta)}$ is the ground state of fluxons (chargeons). $|G\rangle_\phi$ describes the Bose condensate.[17] In the continuum notation,

$$\begin{aligned} \Psi_\phi(x_1,\cdots,x_N) &\equiv {}_\phi\langle 0|\phi_{x_1}\cdots\phi_{x_N}|G\rangle_\phi \\ &= \prod_{i<j}|z_i - z_j|^{2q}\exp\Big[-\frac{1}{4\ell_\phi^2}\sum_{j=1}^N |z_j|^2\Big], \end{aligned} \quad (16)$$

where z_j's are the complex coordinates of N fluxons, $\ell_\phi = (eB_\phi)^{-1/2}$ $(B_\phi \equiv \langle B_x^{\rm CS}\rangle/e = 4\pi qn/e)$. The CS factor $\exp[2iq\sum\theta_{xy}\phi_y^\dagger\phi_y]$ in eq.(6) produces a phase factor of $|z_i - z_j|^{2q}$, changing $|z_i-z_j|^{2q} \to (z_i - z_j)^{2q}$ in the electron wave function. Thus we have

$$\begin{aligned} &\Psi_e(x_1,\cdots,x_N) \equiv {}_C\langle 0|C_{x_1}\cdots C_{x_N}|G\rangle_C \\ &= \prod_{i<j}(z_i-z_j)^{2q}\ e^{-\sum|z_j|^2/(4\ell_\phi^2)}\cdot {}_\eta\langle 0|\eta_{x_1}\cdots\eta_{x_N}|G\rangle_\eta. \end{aligned} \quad (17)$$

At $\nu = p/(2pq\pm 1)$, the *uniform* CS field generated by the condensation of fluxons partly cancels uniform $B^{\rm ex}$. Chargeons feel the residual field $\Delta B = B^{\rm ex} - B_\phi = \pm 2\pi n/(ep)$, and fill the p Landau levels of ΔB, giving rise to IQHE. This observation obviously implies that the chargeons are nothing but Jain's CFs.[1] The wave function of η in eq.(17) is known for $p=1$ as the Slater determinant,

$$_\eta\langle 0|\eta_{x_1}\cdots\eta_{x_N}|G\rangle_\eta = \prod_{i<j}(z_i - z_j)\ e^{-\sum|z_j|^2/(4\ell_\eta^2)}, \quad (18)$$

where $\ell_\eta = (e\Delta B)^{-1/2}$. Thus eq.(17) becomes just the Laughlin's wave function for $\nu = 1/(2q+1)$. (Note that $\ell^{-2} = \ell_\phi^{-2} + \ell_\eta^{-2}$.) For $p \neq 1$, one needs the wave function of IQHE.

At $\nu = 1/(2q)$ ($p = \infty$), $\Delta B = 0$, i.e., the uniform CS field generated by the fluxon condensate completely cancels out $B^{\rm ex}$, thus chargeons behave as quasi-free fermions in zero magnetic field. Beyond the MFT, fluctuations of A_{xj} mediating attractive interaction between chargeon and fluxons may generate non-fermi-liquid behaviors.

Let us consider the EM transport properties of the PFS state. The response functions of electrons are calculated from the effective action $S_{\rm EM}$ defined by

$$\int [dU] \exp(-S_{\rm eff}[a_{xj}, U_{xj}]) = \exp(-S_{\rm EM}[a_{xj}]). \quad (19)$$

In the PFS states, fluctuations of the dynamical gauge field A_{xj} are small, so $S_{\rm eff}[a_{xj}, U_{xj}]$ can be expanded in powers of A_{xj} up to $O(A^2)$ as

$$S_{\rm eff}[a_{xj}, U_{xj}] = \sum_{x,y,i,j} \Big[(A+cea)_{xi} \Pi^{ij}_{\phi;xy} (A+cea)_{yj} + (A+(1-c)ea)_{xi} \Pi^{ij}_{\eta;xy} (A+(1-c)ea)_{yj} \Big], \quad (20)$$

where $\Pi^{ij}_{\phi(\eta)}$ is the polarization tensor of $\phi_x(\eta_x)$. $S_{\rm EM}[a_{xj}]$ is obtained by integrating over $A_{xj} (\in \mathbf{R})$ as

$$S_{\rm EM}[a_{xj}] = e^2 \sum a_{xi} \Pi^{ij}_{xy} a_{yj}, \quad \Pi = (\Pi_\phi^{-1} + \Pi_\eta^{-1})^{-1}, \quad (21)$$

where Π is nothing but the response function of electrons. Then, we obtain the formula for the resistivity,

$$\rho = \rho_\eta + \rho_\phi, \quad (22)$$

where $\rho = (e^2\Pi)^{-1}$, ρ_η, and ρ_ϕ are the 2×2 *resistivity tensor* of electrons, chargeons, and fluxons, respectively.

The formula (22) does not depend on c of eq.(9). In fact, c expresses arbitrariness in choosing the reference state from which the relative EM charges (9) are measured.[18] A similar formula for ρ is known for high-T_c cuprates as the Ioffe-Larkin formula.[19]

What is the contribution to the electric transport from the fluxons? In the CB theory for the FQHE,[2,3,4] each CB carries $2q+1$-flux quanta and gives rise to $\rho_{xy} = (2q+1)h/e^2$. The fluxons in the present formalism certainly contribute to ρ_{xy} as do the CB, hence $\rho_{\phi\, xy} = 2qh/e^2$. Likewise, $\rho_{\phi\, xx} = 0$ because of the superfluidity of the fluxon condensate. On the other hand, the chargeons fill up the p Landau levels of ΔB to contribute with $\rho_{\eta\, xy} = \pm h/(pe^2)$, $\rho_{\eta\, xx} = 0$ as in the IQHE. Thus, from eq.(22), we obtain

$$\rho_{xy}\frac{e^2}{h} = 2q \pm \frac{1}{p} = \frac{1}{\nu}, \quad \rho_{xx} = 0, \quad (23)$$

which are actually observed in the experiments. At $\nu = 1/(2q)$, as a result of the condensation of fluxons, $\Delta B = 0$ and the chargeon behaves as a Fermi liquid. Therefore, $\rho_{xy} = \nu^{-1}(h/e^2)$ and $\rho_{xx} \neq 0$. We shall discuss more details of the physics of quasi-excitations.[20]

Finally, we comment on the role of Coulomb interaction. Its short-range part enhances $T_{\rm PFS}$. In fact, eq.(14) shows that $T_{\rm PFS}$ increases for larger g (smaller γ) if $V_0(T)$ depends on T weakly. On the other hand, the long-range part of Coulombic interaction may renormalize various effective parameters such as the mass of chargeon and the strength of repulsive interactions of fluxons, just as in a conventional Fermi-liquid theory.

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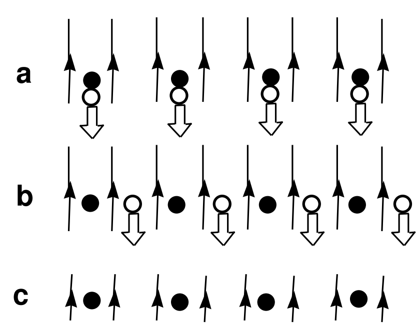

Fig. 1. Illustration of PFS. (a) Electrons in magnetic field $B^{\rm ex}$. Thin arrows are $B^{\rm ex}$, black circles are chargeons η_x, white circles are fluxons ϕ_x, and thick white arrows are CS fluxes. See eq.(6). (b) In PFS states, chargeons and fluxons dissociate. (c) In FQHE states, fluxons form Bose condensate and the resulting uniform CS field cancels $B^{\rm ex}$ partly. Chargeons feel the residual field ΔB (thin arrows).